At Crossroads: Probabilities in everyday Life and Decisions.

Everyday, people make different sorts of decisions - from deciding what to eat, what to wear to what route to take to work or school, based on available information. Most times, the information available is subject to uncertainty, often incomplete and subject to conditions not under our control. Sometimes, our decisions are *binary* – meaning one choice or the other; and other times, there is an array of possibilities. An example of a binary decision is whether to attend an event or not. A non-binary decision may be what route to take to the event (or even how to map out an efficient route), based on what traffic conditions may be, in addition to externalities, such as unforeseen vehicle breakdowns or crashes.

While many events either happen or do not happen, what typically weighs heavily on the decision are the likelihoods of the event. These likelihoods, also called chances, are typically referred to as *probabilities* in the well-known and widely studied field of Mathematics. The field of *probability and stochastic* (or random) processes is an area in Mathematics with pragmatic and crosscutting applications.

Businesses that hedge bets on risks (for instance, insurance companies) often need to make informed guesses based on available information and uncertainties they are contending with. Based on weather patterns and meteorology data, meteorologists make weather predictions. Experts in the financial industry, for instance, the stock markets have to predict market trends based on several factors -- or actually their likelihoods.

In studying probabilities and decision making amidst uncertainty, some notions are important - independence, mutual exclusivity and conditional probability. We start out by briefly and gently introducing the concept of independence in decision making or estimating likelihoods of events. One important thing to note is that probabilities lie between the numbers 0 and 1. When it is zero, it means that the event is not just unlikely, but not going to happen and when it is 1, it means that the event is almost guaranteed to happen.

Independence

To explain the concept of independence, let us assume some woman named Mrs. Glory Olatuwabo is pregnant. Now, suppose that you and one of your friends are having a conversation, trying to guess the gender of the baby. It is possible that you and your friend could be making all kinds of argument as to why the baby will be boy or girl.[1] If you, however, consider the fact that the outcome could be either gender

[1] We are excluding the possibility of mixed genders and other unusual birth complications in this illustrative example.

with equal likelihood, the conversation with your friend becomes easier to navigate. At the end of the conversation, you and your friend are likely to hit a dead end if the notion of *independence of events* is not taken into account. Many would agree that the chances (or probability) of the gender of the baby being a boy or girl is equally $\frac{1}{2}$, since there are two outcomes, with each equally likely.

Suppose we add another dimension (or additional information) to the conversation, how does it affect the outcome of the conversation? In particular, let us assume that you had some side information that the expected baby is Mrs. Olatuwabo's fifth child and her first four children were 3 girls G and 1 boy B (perhaps in the following order $G\ B\ G\ G$. Does the $\frac{1}{2}$ probability of the baby being a boy or girl change in making an informed guess of the baby's gender?

If you guessed 'no, it does not change', then you are right! The underlying assumption is that the gender of each conceived baby (or birth) is **independent** of the other. This is similar to tossing a coin. When a fair[2] coin is tossed, the outcomes are either a Head H or Tail T. The likelihood of the outcome H or T for each toss is $\frac{1}{2}$, and the tosses are considered *independent*. Let us apply this understanding to another slightly more involved problem, which will use and illustrate the *value of information* in determining probabilities. The concept of independence in probability and decision-making helps people realize how unrelated certain events are. It is also slightly related to the concept of *mutual exclusivity*. In brief, the notion of mutual exclusivity explains the fact that two (exclusive) outcomes are not possible. Let us briefly see this in play on the conversation about Mrs. Olatuwabo's baby. Since the gender of the baby will either be a boy B or girl G, and not both, the outcomes are mutually exclusive. That is, the gender cannot be both[3].

Now back to independence. To illustrate the value of information and how it affects the probability of an event, let us consider another interesting example of a decision problem that Dr. Paul Ebiturbo has to make. Since this is slightly more involved, we will likely introduce placeholders, called *variables* as we carry out simple mathematical computations. Variables typically represent probabilities of an outcome, which one may not want to express numerically. For instance, when a fair 6-sided dice is rolled, since the outcomes are either a $1,\ 2,\ 3,\ 4,\ 5,$ or 6, the probability that the outcome is even is $\frac{3}{6}$, which can be simplified to $\frac{1}{2}$. Rather than explicitly express those probabilities in numbers, we represent them as variables. As an example on the roll of the fair dice, one can say, let the variable x represent the probability that the outcome

[2] By a fair coin, we mean a two-sided coin.
[3] Again, this assumes that we exclude very strange and rare situations where the baby may somehow have organs that in some way, shape or form indicates *bi-gender*.

is even and the variable y represent the probability that the outcome is odd. Variables are important in mathematics and should be interpreted as 'place holders'.

Hunting decisions

Dr. Alikor, a physician and hunter in rural Bayelsa, has two Beagles. When out hunting one day, on the trail of an animal, Dr. Alikor arrives at a place where the road diverges into two paths. He knows that each Beagle, independent of the other, will choose the correct path with some probability p.[4] Dr. Alikor decides to let each dog choose a path, and if they agree, he will take their decision; however, if they disagree, he will pick a path at random. The task here is to figure out **whether Dr. Alikor's strategy is better than letting one of the two Beagles decide on a path**? It would help to illustrate our thinking in terms of p (that is, the probability of each Beagle independently choosing the correct path).

To address this problem, let us consider the sample space (that is, the set or space of possible outcomes) for Dr. Alikor. First, we will start by articulating and outlining the events that lead to choosing the correct path. In particular, Dr. Alikor chooses the correct path based on one of the following outcomes:

1. First, both beagles agree on the correct path. Since both Beagles choose the correct path with probability p *independent* of the other, if both beagles agree, it means that the first beagle chose correctly *and* the second beagle chose correctly. That is, p and p, which yields $p \times p = p^2$, by independence.
2. Second, the beagles disagree, and the first beagle chooses the correct path, and Dr. Alikor follows the first beagle with probability $p \times \frac{1-p}{2}$. Let us break this down. Since probabilities of potential outcomes sum to 1, and we know that the first beagle chose the correct path with probability p, it implies the second beagle chose the incorrect path with probability $1 - p$. Since they happen in sequence, we have the product $p \times (1 - p) = p(1 - p)$. Dr. Alikor picks a path from one of the two at random (his probability of picking any path is equally likely, hence he picks with probability $\frac{1}{2}$). Again, since these happen in sequence, the probability of this outcome is $p \times 1 - p \times \frac{1}{2} = p(1 - p)/2$.
3. The third possible outcome is that the beagles disagree and the second beagle chooses the correct path, and Dr. Alikor follows the second beagle with probability $p\ (1 - p)/2$. A similar explanation can be given for this probability.

[4] A correct path in this example is one that results in Dr. Alikor hunting down some bush meat for his lovely wife.

The above events are disjoint, so we can add up the probabilities to find that under Dr. Alikor's strategy, the probability that he chooses the correct path is

$$p^2 + \frac{1}{2}\, p\,(1-p) + \frac{1}{2}\, p\,(1-p) = \; p.$$

On the other hand, if Dr. Alikor lets one beagle choose the path, this beagle will also choose the correct path with probability p. Thus, the two strategies are equally effective.

As a gentle mental exercise, we encourage the reader to ponder on why the second and third probability expressions are alike.